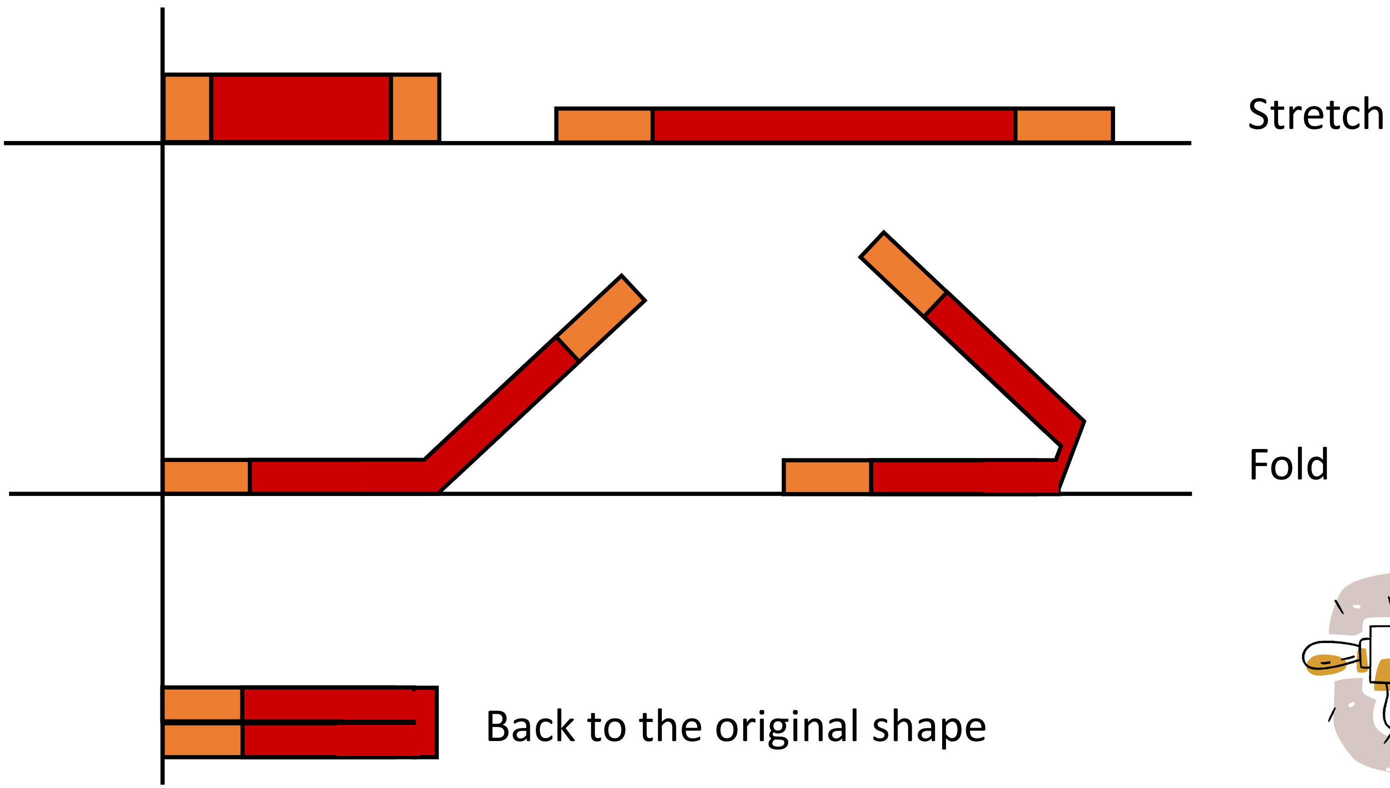
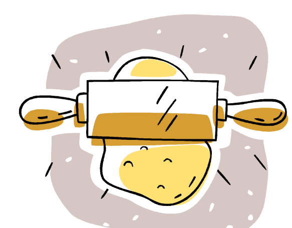
Stretch and Fold

With layers

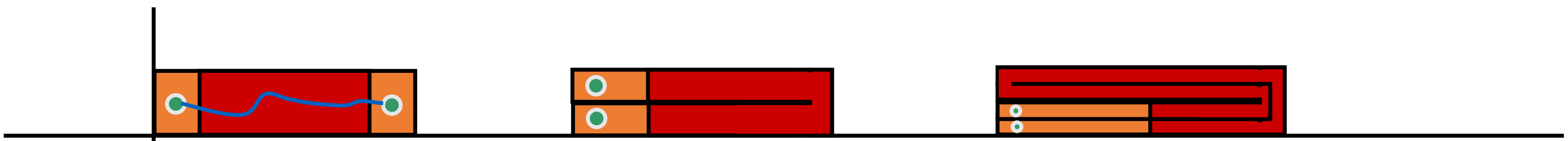




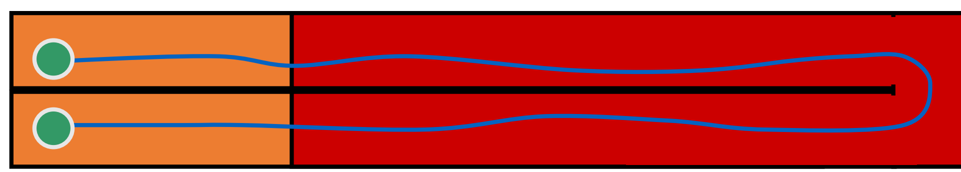
Slides from Frederick H. Willeboordse

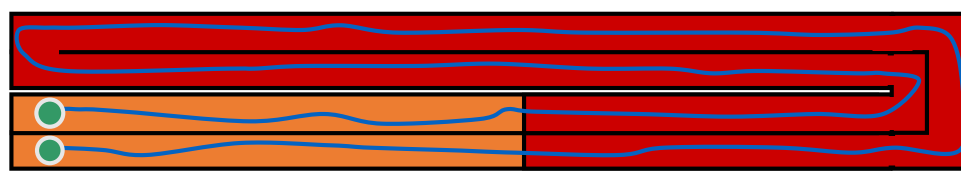
Stretch and Fold

The distance between points on opposite end of the bar.



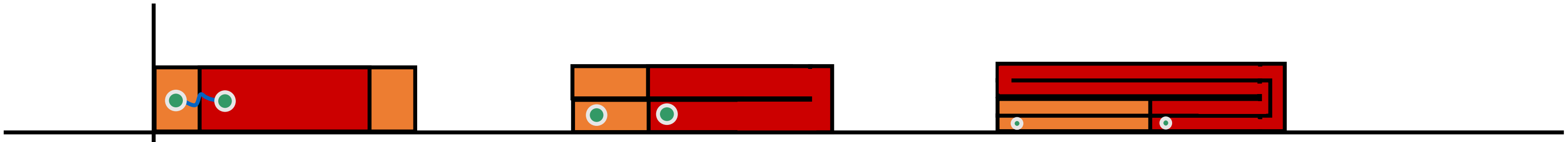
The distance grows exponentially!



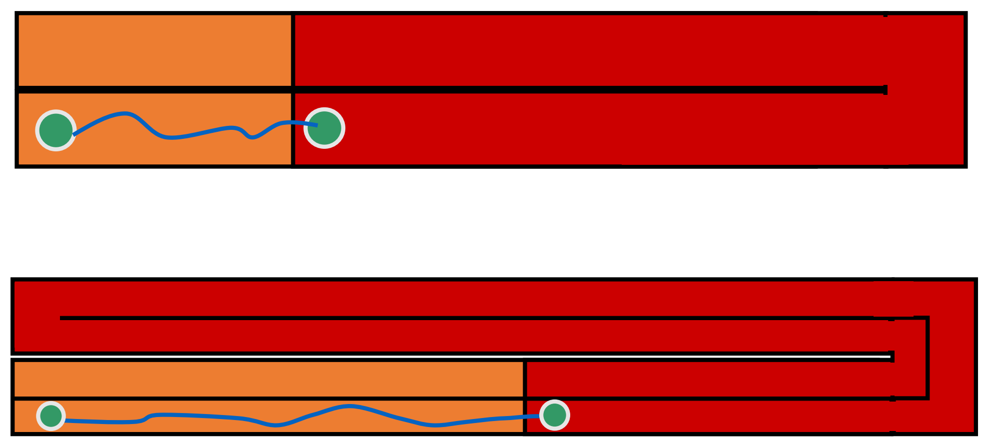


Stretch and Fold

The distance between nearby points. Sensitive Dependence!



The distance grows exponentially!

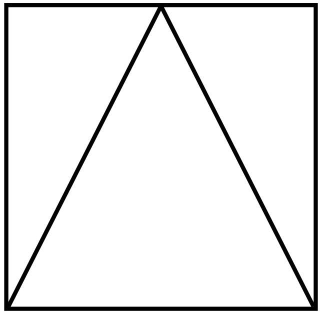


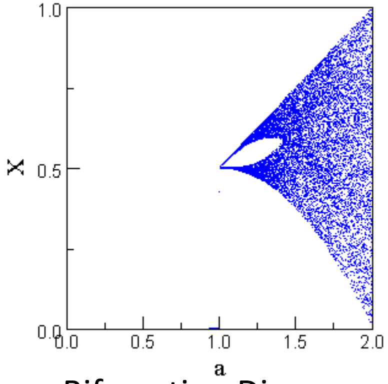
Stretch and Fold

Mathematically

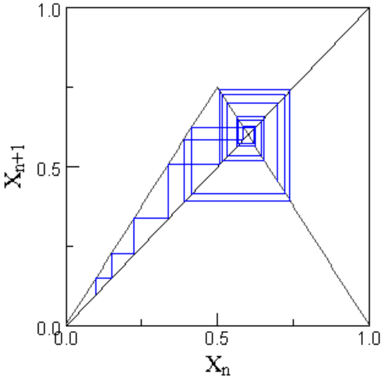
$$T(x) = \begin{cases} ax & \text{if} \quad x \le 0.5 \\ -ax + a & \text{if} \quad x > 0.5 \end{cases}$$

The Tent Map

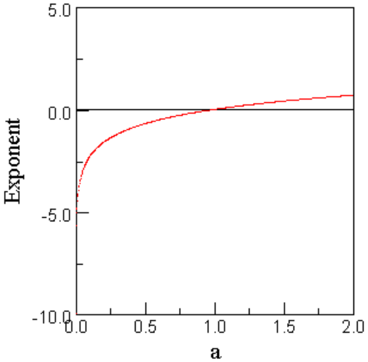




Bifurcation Diagram



Cobweb



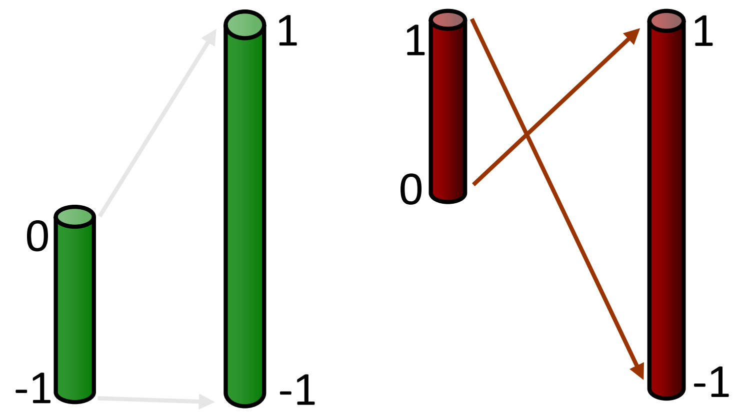
Lyapunov Exponents

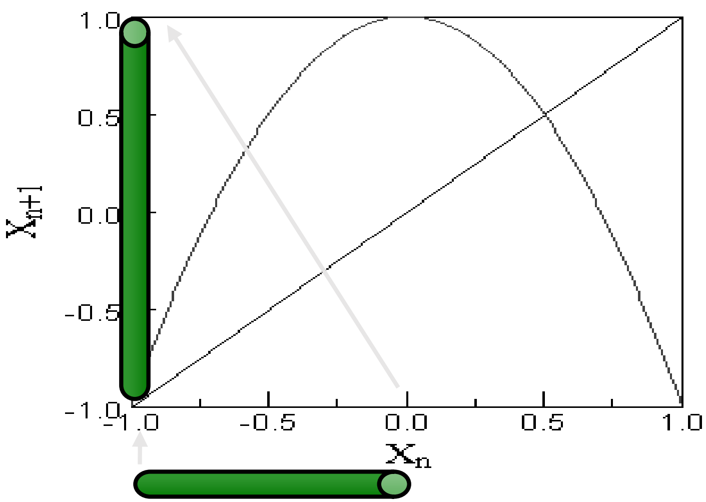
https://en.wikipedia.org/wiki/Horseshoe_map

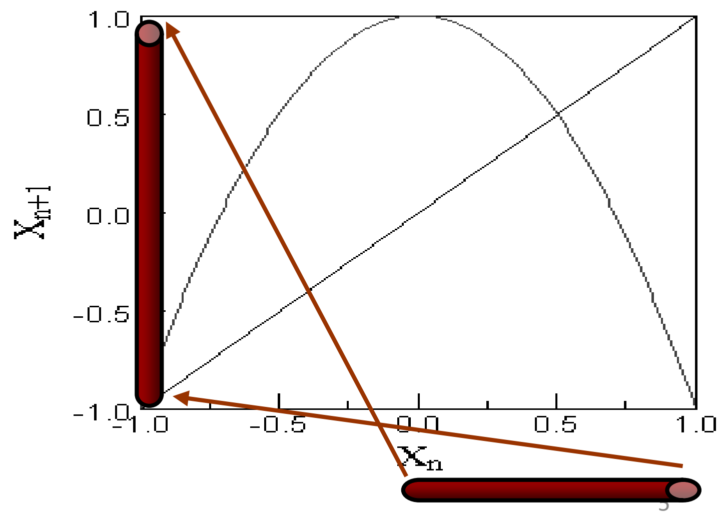
Stretch and Fold

In the logistic map

The same as stretch and fold with the stretch being nonlinear.





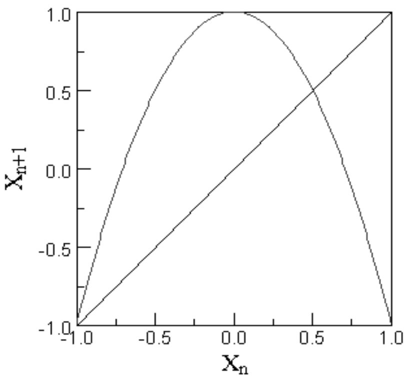


Universality

A key motivation for the study of chaos is the notion of ***universality***. In this context it means that a certain feature or a certain constant is applicable to a whole range of systems which are said to be a ***class of systems***.

It is important to note that universality in this sense does not mean everywhere in all conceivable cases.

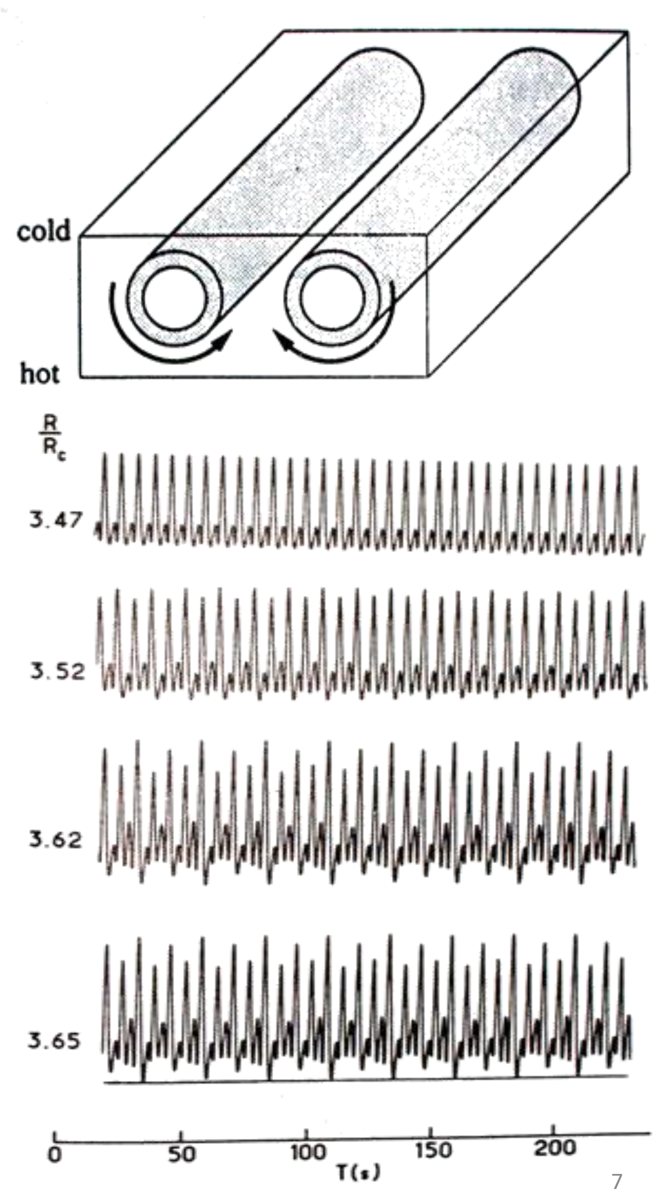
The most well known universal constant in chaos theory is the ***Feigenbaum constant***. It applies to all single hump functions.



Universality

Some Examples

Experimental verifications of the Feigenbaum constant.



Experiment	Number of period doublings	δ	Authors
Hydrodynamic			
water	4	4.3 (8)	Giglio et al. (1981)
mercury	4	4.4 (1)	Libchaber et al. (1982)
Electronic			
diode	4	4.5 (6)	Linsay (1981)
diode	5	4.3 (1)	Testa et al. (1982)
transistor	4	4.7 (3)	Arecchi and Lisi (1982)
Josephson simul.	3	4.5 (3)	Yeh and Kao (1982)

The chaotic behavior of the leaky faucet

P.MartienS.C.PopeP.L.ScottR.S.Shaw (1985)

Chaos in a dripping faucet

H N Nunez Yepez, A L Salas Brito, C A Vargas and L A Vicente

European Journal of Physics, Volume 10, Number 2 (1989)

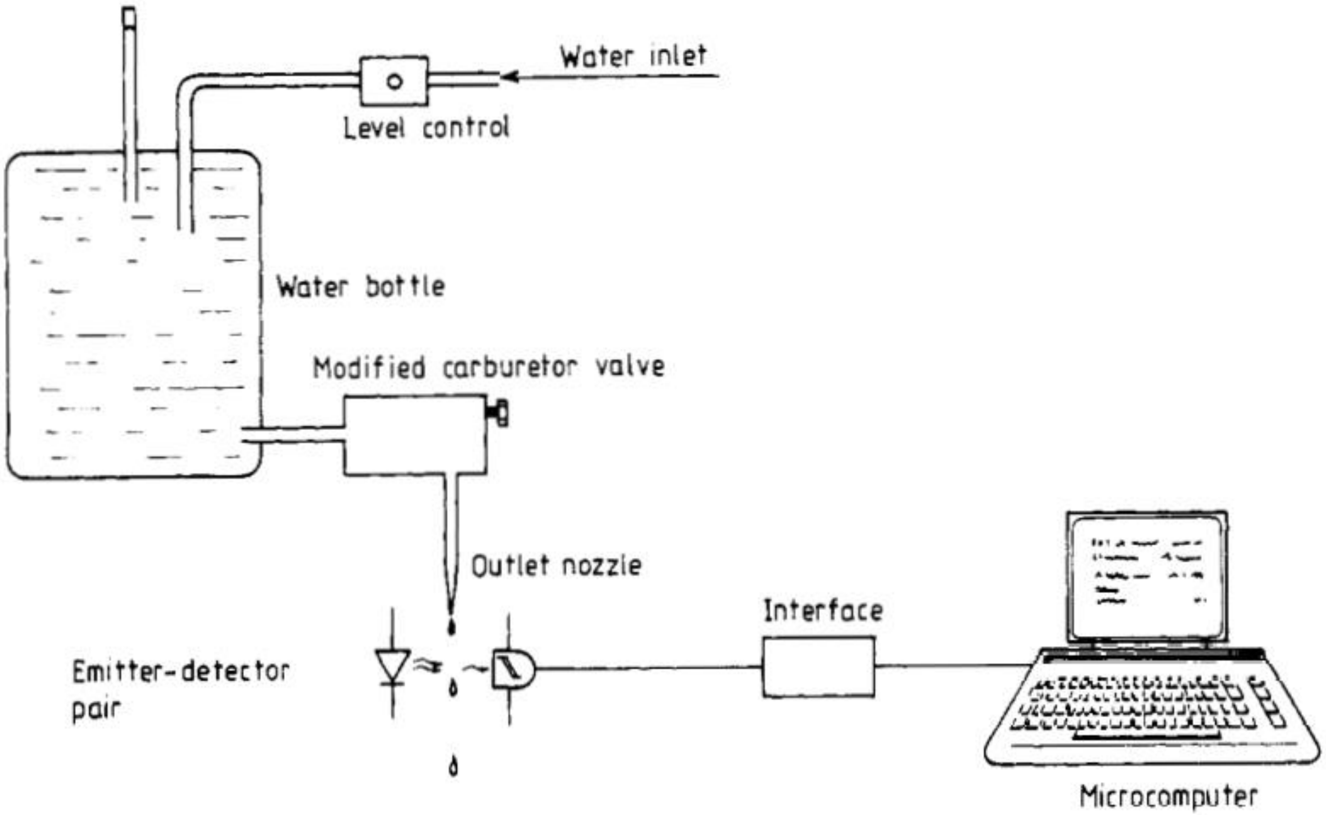
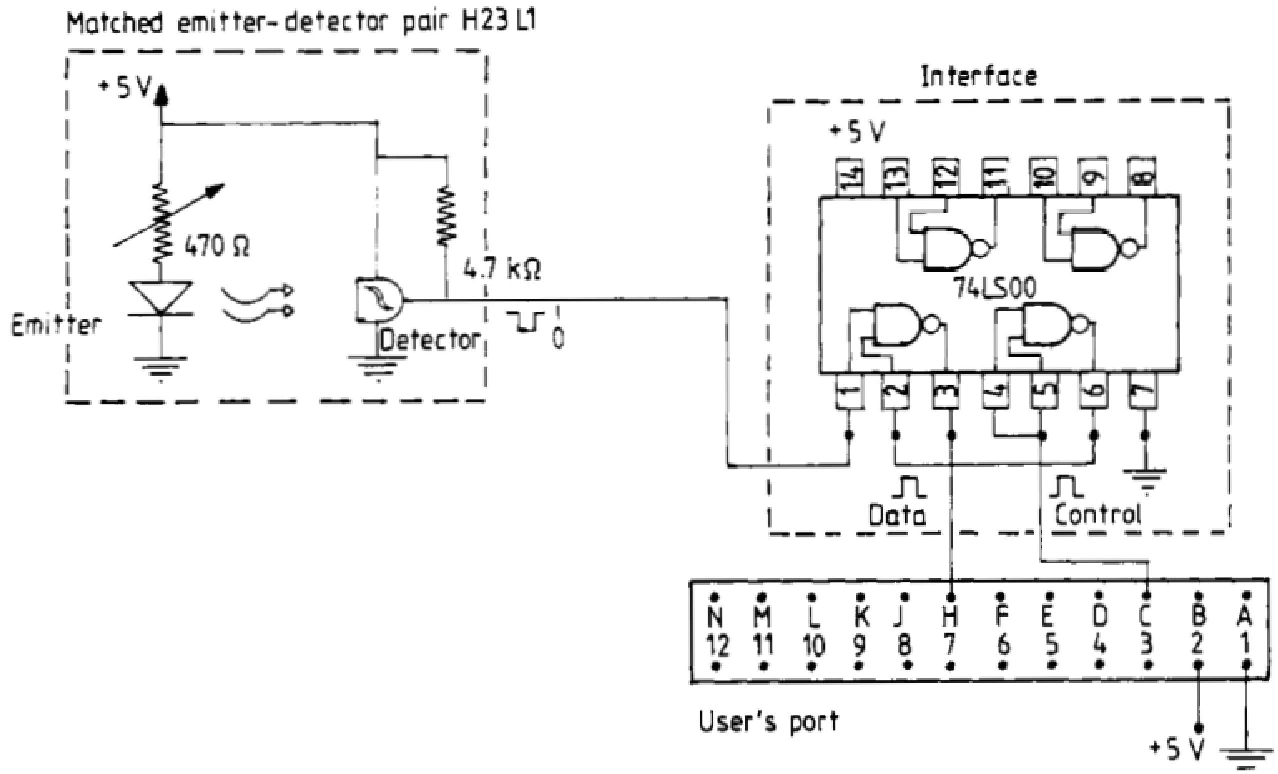


Figure 3. Schematic diagram of the experimental set-up. We use a float valve (marked 'level control' in the diagram) to maintain the water level in an upper reservoir (not shown).

Figure 4. The interface is a single 74LS00 chip. The connections to the microcomputer user's port are shown.



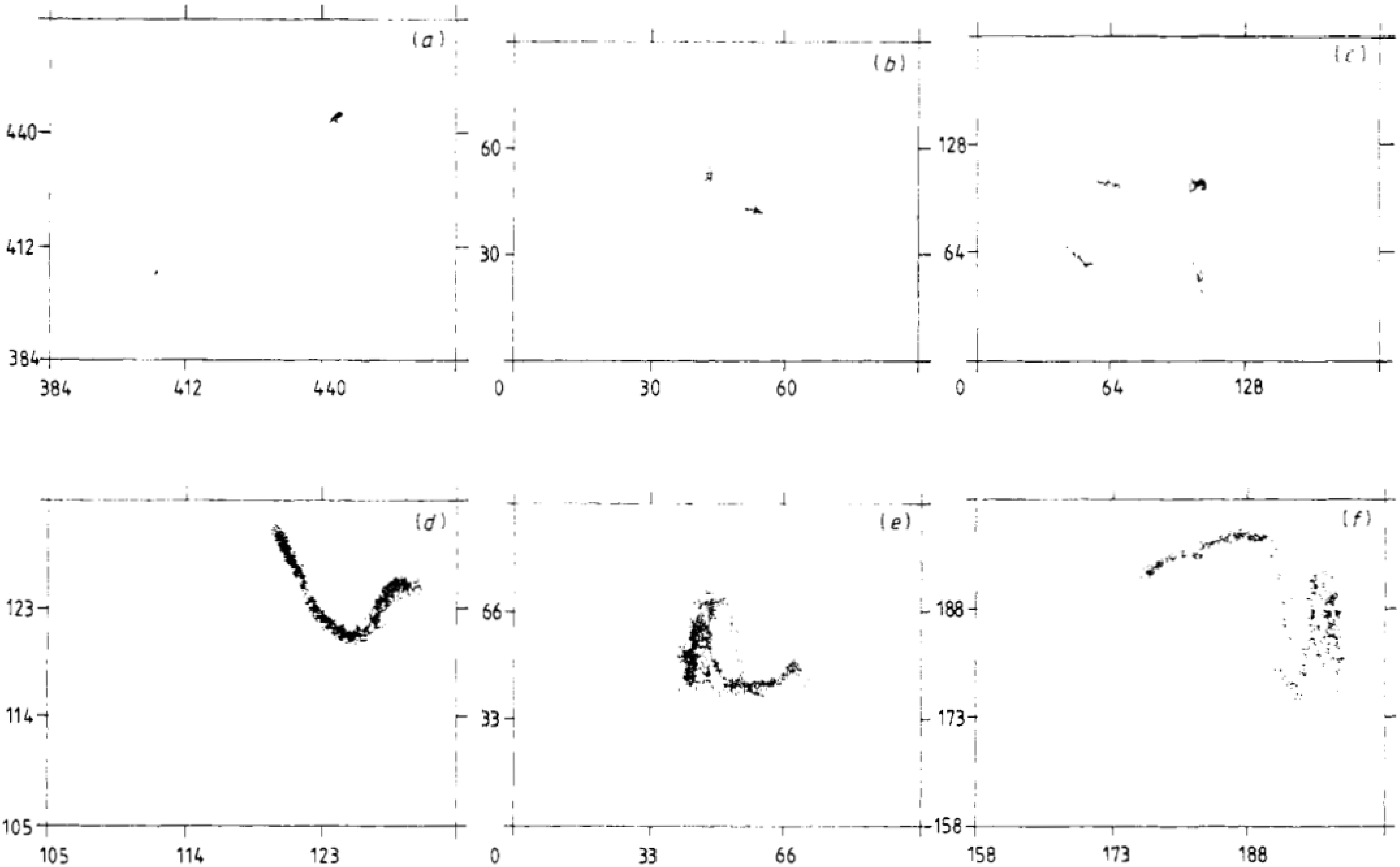


Figure 5. Example of the experimental results shown as T_{n+1} (vertical axes) versus T_n (horizontal axes) graphs redrawn from the printout of our data. Periodic behaviour, (*a*)–(*c*); complex chaotic behaviour, (*d*)–(*f*). All values of time are in milliseconds.