Atomic motion in a magnetic trap

Computer simulations in physics

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1 Introduction

Magnetic trapping of neutral atoms has the potential for use in many areas, including high-resolution precession spectroscopy, Bose-Einstein condensation and quantum optics.
In traps, coils create a magnetic field that, due to its geometry, directs neutral atoms to a certain location. Confinement of neutral atoms depends on an interaction between an inhomogeneous electromagnetic field and an atomic multipole moment.
In addition to my undergraduate studies, I work as a student at Quantum Optical Research Group at the Wigner Research Centre, where they study the interaction between trapped atoms and light. For light-atom interaction, may depend upon knowing both atoms positions and velocities, so that laser beams of proper polarization and direction can be applied.
With the chosen task, I create a simple simulation of the motion of atoms in the magnetic field of the trap there.

2 The problem and solution

In the chosen project, I define the motion and the trajectory of an atom in the trap. The task thus consists of two parts, first the calculation of the magnetic field of the trap used in the laboratory, and then the solution of the equations of motion of the atom interacting with the magnetic field. I use a classical approach to describe the motion of the atom.

2.1 The magnetic field of the trap

Knowing the position and parameters of the coils, the magnetic field can be calculated. To calculate the field, we must start from the Maxwell-equations:

$$\begin{aligned} \bigtriangledown \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \bigtriangledown \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \bigtriangledown \cdot \mathbf{B} &= 0 \\ \bigtriangledown \times \mathbf{B} &= \mu_0\big(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{D}}{\partial t}\big) \end{aligned} \tag{2.1}$$

The equations can be solved analytically, [1] or numerically [3], . In the case of a numerical solution, a PDE system must be solved.

2.2 The motion of an atom

After we have a magnetic field, we can study the motion of an atom in it. Consider the neutral atom as a dipole and write the following Hamiltonian equation of motion:

$$H = \frac{\mathbf{P}^2}{2M} + \underline{m} \cdot \underline{B} \tag{2.2}$$

Where $\mathbf{P} = (P_x, P_y, P_z)$ is the momentum, M is the atomic mass, $\mathbf{m} = p \cdot \mathbf{l}$ is the magnetic dipol moment, depends on two factors: the strength p of its poles (magnetic pole strength), and the vector $\mathbf{l}$ separating them.

I study the motion of the atom in 2 dimensions, in the z = 0 plane, introducing the $\rho = \sqrt{x^2 + y^2}$ radius and $P_\rho = \sqrt{P_x^2 + P_y^2}$. Under these conditions, the motion of the atom satisfies the following system of ordinary differential equations:

$$\begin{aligned} \frac{d\rho}{dt} &= \frac{dH}{dP_\rho} \\ \frac{dP_\rho}{dt} &= -\frac{dH}{d\rho} \end{aligned} \tag{2.3}$$

An adaptive Runge-Kutta method can be used to solve the following ODE system. After the solution, I can plot the orbit of the atom in the magnetic field with the obtained position, momentum and time data, [1, 3].

3 The plan

I plan to calculate the magnetic field of the trap used in the lab and then determine the orbit of the trapped atom. I do this for different initial conditions, examining, on the one hand, the motion of the atom and, on the other hand, how long it remains trapped.

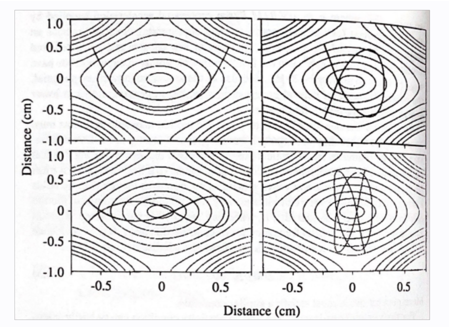

Figure 1: An example for orbits of an atom in a magnetic trap, [3].

References

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[2] Rubin H. Landau, Manuel J. Páez, Cristian C. Bordeianu, A Survery of Computational Physics, 2012

[3] Harold J. Metcalf, Peter van der Straten, Laser Cooling and Trapping